pages	1 − 2(15)	3 − 4(12)	5 − 6(8)	7 − 9(9)	10 − 11(3)	12 − 13(18)	14 − 15(8)	*total*(73)
scores								

Exam #2, April 13, Linear Algebra (201), Spring, 2018, W. Stephen Wilson

Name : ______________________________

TA Name and section: ______________________________

NO CALCULATORS, NO PAPERS, SHOW WORK. PUT YOUR ANSWERS IN BOXES! .

We begin with modified True/False questions. Zero points for getting the wrong answer. One point for leaving it blank. 2 points for getting the correct answer, and a 3rd point for giving a very short correct explanation for your correct answer. You cannot get this point of your T/F answer is incorrect.

1. (3 points) The map $T : P_2 \longrightarrow \mathbb{R}$, $T(f(x)) = \int_0^1 f(x)dx$ is a linear transformation.

2. (3 points) There exists a linear transformation from $P_6 \longrightarrow \mathbb{R}$ whose kernel is isomophic to $\mathbb{R}^{2\times 2}$.

3. (3 points) We have a linear space V. We have a linear transformation $T : V \longrightarrow V$. We have a basis $\mathscr{B}$ for V. The matrix for T with respect to the basis $\mathscr{B}$ is $\begin{pmatrix} 3 & 5 \\ 0 & 4 \end{pmatrix}$. There is an element $0 \neq x \in V$ such that $T(x) = 5x$.

4. (3 points) There exists a subspace $V \subset \mathbb{R}^5$ such that $dim(V) = dim(V^{\perp})$.

5. (3 points) If $n \times n$ matrics A and B commute, then matrices A^T and B^T must commute as well.

6. (3 points) If V is a suspace of $\mathbb{R}^n$ and $\vec{x} \in \mathbb{R}^n$, then the inequality $\vec{x} \cdot (proj_V \vec{x}) \geq 0$ must hold.

7. (3 points) There exist invertible 3×3 matrices A and S such that $S^T A S = -A$.

8. (3 points) If A is a skew-symmetric 4×4 matrix ($A^T = -A$), then the determinant of A is zero.

9. (3 points) If the determinant of a 5×5 matrix is 5, then its rank must be 5.

End True/False

10. (3 points) Let $M_{2\times 2}$ be the 2×2 matrices with basis $\mathscr{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$. Let $S = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Define $L : M_{2\times 2} \longrightarrow M_{2\times 2}$ as $L(A) = S^T AS$. Find the 4×4 matrix for L with respect to the basis $\mathscr{B}$.

11. (3 points) What is the dimension of the kernel for the previous L and why?

12. (2 points) What is the dimension of the image for the previous L and why?

Let $M^{sk}_{3\times 3}$ be the 3×3 skew-symmetric matrices ($A^T = -A$) with basis
$\mathscr{B} = \left\{ \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \right\}$ and inner product $< A, B >= tr(A^T B)$ where tr is the trace, i.e. the sum of the diagonal elements of a square matrix. Be careful. This sequence of problems probably requires getting each previous problem correct. Use good quizmanship.

13. (3 points) Find an orthonormal basis for the subspace, V, spanned by $\left\{ \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \right\}$.

There is another page for work.

This page for work.

14. (3 points) Compute the orthogonal projection of $\left\{ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \right\}$ in V.

15. (3 points) Compute the 3×3 matrix for the orthogonal projection of $M^{sk}_{3\times 3}$ to V with respect to the above basis $\mathscr{B}$.

16. (3 points) Consider the linear transformation (take my word for it that it is a linear transformation that goes to and from the right places) $T : M^{sk}_{3\times 3} \longrightarrow M^{sk}_{3\times 3}$ given by:

$$T(A) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} A \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Compute the 3×3 matrix for T using the above basis $\mathscr{B}$. There is another page for work.

This page for work.

17. (3 points) What is the area of the parallelogram in $\mathbb{R}^3$ made by the two vectors $(1, 2, -1)$ and $(2, 1, -2)$.

18. (3 points) What is the cosine of the angle between the two vectors $(1, 2, -1)$ and $(2, 1, -2)$.

19. (3 points) What is the volume of the 3-dimensional object defined by vectors e_1, e_2, and e_3?

20. (3 points) What is the volume of the 3-dimensional object defined by vectors $(1, 2, -1)$, $(2, 1, -2)$, and $(3, 0, -1)$

21. (3 points) If the determinant of the matrix with columes u_1, u_2, and u_3 is 3, what is the determinant of the matrix with the columns $(u_2 + u_3, 2u_3, u_2 + u_1)$?

22. (3 points) If the $m \times n$ matrix $B = (u_1, u_2, \ldots, u_n)$ has orthonormal columns u_k, and $A = 5B$, what is the determinant of the $n \times n$ matrix $A^T A$.

I'm only giving 2 points per problem on this last sequence of problems. I don't recommend you doing them unless you feel confident about the previous problems that you thought you could work. This is a long winded sequence of problems related to the (assumed) linear transformation $T : P_2 \longrightarrow P_2$ given by $T(f(x)) = f(1-x)$. I give you two bases, $\mathscr{A} = \{x^2, x, 1\}$ and $\mathscr{B} = \{(x-1)^2, (x-1), 1\}$. Be careful. If f is the functon 1, what is $f(t)$?

23. (2 points) What is the 3×3 matrix C such that $C[x]_{\mathscr{B}} = [T(x)]_{\mathscr{A}}$.

24. (2 points) What is the 3×3 matrix A such that $A[x]_{\mathscr{A}} = [T(x)]_{\mathscr{A}}$.

25. (2 points) What is the 3×3 matrix B such that $B[x]_{\mathscr{B}} = [T(x)]_{\mathscr{B}}$.

26. (2 points) What is the 3×3 matrix S such that $S[x]_{\mathscr{B}} = [x]_{\mathscr{A}}$.