pages	$1-3(5)$	$4-6(5)$	$7-8(5)$	$9-10(6)$	$11-12(6)$	$13(7)$	*total*(34)
scores							

Final Exam, May 9, Linear Algebra (201), Spring, 2018, W. Stephen Wilson

Name : ______________________________

TA Name and section: ______________________________

NO CALCULATORS, NO PAPERS, SHOW WORK .

Let $v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ give a basis $\mathscr{A} = \{v_1, v_2\}$ for a subspace $V \subset \mathbb{R}^3$.
Let $A = (v_1, v_2)$ be the matrix with columns v_1 and v_2.
We will use these for much of the exam.

1. (2 point) Apply the Gram-Schmidt process to v_1 and v_2 to get an orthonormal basis, $\mathscr{B} = \{u_1, u_2\}$ for V.

2. (1 point) What is the 3×3 matrix $B : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ that is orthogonal projection to V with respect to the standard basis.

3. (1 point) Find an $0 \neq x \in \mathbb{R}^3$ that is perpendicular to V.

4. (1 point) Using the basis $\mathscr{C} = \{v_1, v_2, x\}$ where x is the vector from the previous problem, find the 3×3 matrix $B : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ that is orthogonal projection to V with respect to the basis $\mathscr{C}$.

5. (2 point) Write $A = QR$ where Q is a 3×2 matrix with orthonormal columns and R is a 2×2 upper triangular matrix.

6. (1 point) Find the least squares solution for the equation $Ax = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

7. (2 point) What are the $\mathscr{A}$-coordinates of $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ in V? And the $\mathscr{B}$-coordinates?

8. (3 point) Find an $0 \neq x \in V$ that is perpendicular to $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. First in standard basis coordinates for $\mathbb{R}^3$ and then in $\mathscr{A}$ coordinates in V and in $\mathscr{B}$ coordinates in V.

9. (2 point) Let $L \subset V \subset \mathbb{R}^3$ be the line spanned by $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. Find the 2×2 matrix using $\mathscr{A}$-coordinates for orthogonal projection $V \longrightarrow L \subset V$. Find the 2×2 matrix using $\mathscr{B}$-coordinates for orthogonal projection $V \longrightarrow L \subset V$.

10. (2 point) Find the Eigenvalues for $A^T A$.

11. (2 point) What are the singular values associated with $A^T A$?

12. (2 point) Find the associated orthonormal Eigenvectors for $A^T A$. Call them v_1 and v_2, distinct from the v_i we have been using so far.

13. (3 point) Find orthonormal u_1, u_2, u_3 (not the same u_1, u_2 we've been using, but, yes, you need u_3 here) such that $Av_1 = \sigma_1 u_1$ and $Av_2 = \sigma_2 u_2$

14. (3 point) Give the singular value decomposition of A, i.e. find 3×3 U, 3×2 Σ and 2×2 V^T such that $A = U\Sigma V^T$ where U and V are orthogonal matrices.

15. (1 point) What matrix B gives the quadratic form $q(x) = 2x_1^2 + 2x_1x_2 + 2x_2^2$ as $x^T Bx$?

16. (4 point) Find an orthonormal basis $\mathscr{C} = \{u_1, u_2\}$, λ_1, λ_2 such that the above $q(x) = q(c) = \lambda_1 c_1^2 + \lambda_2 c_2^2$ when using the coordinates in the basis $\mathscr{C}$.

17. (2 point) Set the above $q(x) = 1$. Find coordinates for a point closest to the origin in standard basis form and using $\{u_1, u_2\}$-coordinates.