Homework for 3/3

Some definitions

Consider the set $\mathcal{C}(X,Y)$ of continuous functions from $X$ to $Y$. For every $C \subset X$ compact and every $U \subset Y$, consider the set

$$S(C,U) = \{ f \mid f \in \mathcal{C}(X,Y), \ f(C) \subset U \}$$

Let the subsets $S(C,U) \subset \mathcal{C}(X,Y)$ form a subbasis for a topology on $\mathcal{C}(X,Y)$ called the compact-open topology.

Define $e : X \times \mathcal{C}(X,Y) \rightarrow Y$ by $e(x,f) = f(x)$.

For $f : X \times Z \rightarrow Y$ continuous define $F : Z \rightarrow \mathcal{C}(X,Y)$ by $F(z)(x) = f(x,z)$.

For $A \subset X$, $(X,d)$ a metric space, we define the distance from $x$ to $A$ by $d(x,A) = \inf \{ d(x,a) \mid a \in A \}$.

Define the $\delta$-neighborhood of $A$ in $X$ to be the set $U(A,\delta) = \{ x \mid d(x,A) < \delta \}$.

Let $\mathcal{H}$ be the collection of all nonempty closed, bounded subsets. For $A, B \in \mathcal{H}$, define

$$D(A,B) = \inf \{ \epsilon \mid A \subset U(B,\epsilon) \ \text{and} \ B \subset (A,\epsilon) \}$$

end definitions

Homework problems

1. Let $X$ be a metric space. Suppose that for some $\epsilon > 0$, every $\epsilon$-ball in $X$ has compact closure. Show that $X$ is complete.

2. Let $X$ be locally compact Hausdorff. Show that $e$ above is continuous.

3. Show $D$ above is a metric on $\mathcal{H}$. 