Homework for 3/3

Some definitions

Consider the set $\mathcal{C}(X,Y)$ of continuous functions from X to Y. For every $C \subset X$ compact and every $U \subset Y$, consider the set

$$S(C,U) = \{f \mid f \in \mathcal{C}(X,Y),\ f(C) \subset U\}$$

Let the subsets $S(C,U) \subset \mathcal{C}(X,Y)$ form a subbasis for a topology on $\mathcal{C}(X,Y)$ called the **compact-open topology.**

Define $e : X \times \mathcal{C}(X,Y) \longrightarrow Y$ by $e(x,f) = f(x)$.

For $f : X \times Z \longrightarrow Y$ continuous define $F : Z \longrightarrow \mathcal{C}(X,Y)$ by $F(z)(x) = f(x,z)$.

For $A \subset X$, (X,d) a metric space, we define the **distance from** x **to** A by $d(x,A) = inf\{d(x,a) \mid a \in A\}$.

Define the δ-neighborhood of A in X to be the set $U(A,\delta) = \{x \mid d(x,A) < \delta\}$.

Let $\mathcal{H}$ be the collection of all nonempty closed, bounded subsets. For $A, B \in \mathcal{H}$, define

$$D(A,B) = inf\{\epsilon \mid A \subset U(B,\epsilon) \quad \text{and} \quad B \subset (A,\epsilon)\}$$

end definitions

Homework problems

1. Let X be a metric space. Suppose that for some $\epsilon > 0$, every ϵ-ball in X has compact closure. Show that X is complete.

2. Let X be locally compact Hausdorff. Show that e above is continuous.

3. Show D above is a metric on $\mathcal{H}$.