Homework for 4/2

1. Let $A \subset \mathbb{R}^n$ and $B \subset \mathbb{R}^m$ be convex. Show $A \times B$ is convex in $\mathbb{R}^n \times \mathbb{R}^m$.

2. Show that a simplicial complex $K \subset \mathbb{R}^n$ is closed.

Our definition of simplicial complex is very geometric, but it can easily be abstracted. An **abstract simplicial complex** K is a set of (finite for us) vertices, V, and a collection of subsets, $\mathcal{W}$, of V. Think of each subset in $\mathcal{W}$ as a simplex and you can easily see how to go back and forth between our geometric version and the abstract version. First, it is easy to abstract the geometric version. Second, to get a geometric version from the abstract version just pick linearly independent vectors in some big $\mathbb{R}^n$, one for each vertex. Fill them in with geometric simplices. The abstract way is easier to think about because you don't have to keep track of all those irritating little points in the geometric version. It is enough to know the vertices and the simplices.

More abstract. Take a finite set, X. Take some collection, $\mathcal{W}$, of (non-empty) subsets. Make a simplicial complex by taking elements of $\mathcal{W}$ to be the vertices and the simplices to be subsets of $\mathcal{W}$ with non-empty intersection. This is getting abstract. This simplicial complex is called the **nerve** of $\mathcal{W}$.

3. Show that the nerve is an (abstract) simplicial complex.

4. Let $X = \{1, 2\}$ and $\mathcal{W} = \{\{1\}, \{1, 2\}\}$. What is the nerve of this?

5. Let $X = \{1, 2, 3\}$ and $\mathcal{W} = \{\{1\}, \{1, 2\}, \{1, 2, 3\}\}$. What is the nerve of this?

6. Let $X = \{1, 2, 3\}$ and $\mathcal{W} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$. What is the nerve of this?

7. Let $L \subset K$ be a sub-simplicial complex of K with vertices of L given by V and those for K by W. Take the set of vertices $W - V$ and all the simplices of K that have only those vertices. Show this is a simplicial complex.