Homework for 4/9

Some definitions

We will start our work with the triangular disk, i.e. the simplicial complex with 3 vertices, 3 line segments joining them (1-simplices) and also the big 2-simplex, so that our triangle is not hollow. (no holes, not the circle)

Let's call that K^0. And now let's call the n-th barycentric subdivision K^n. Let $s_0(n)$ be the number of vertices of K^n, $s_1(n)$ be the number of 1-simplices of K^n and $s_2(n)$ the number of 2-simplices.

Note that for our K^0 we have $s_0(0) = 3$, $s_1(0) = 3$, and $s_2(0) = 1$.

End definitions

1. For starters, draw pictures of the first and second barycentric subdivisions. To check your work, we have $s_0(1) = 7$, $s_1(1) = 12$, $s_2(1) = 6$, $s_0(2) = 25$, $s_1(2) = 60$, and $s_2(2) = 36$

2. Find recursive formulas for $s_0(n)$, $s_1(n)$, and $s_2(n)$. That is, write them all in terms of $s_0(n-1)$, $s_1(n-1)$, and $s_2(n-1)$.

3. Prove that $s_2(n) - s_1(n) + s_0(n) = 1$.

4. Find closed formulas for $s_0(n)$, $s_1(n)$, and $s_2(n)$. That is, we don't want a recursive formula, but something you just plug in n to and get your answer, e.g. $s_2(n) = 6^n$ (in case you hadn't already noticed).