Homework for 4/9

Some definitions

We will start our work with the triangular disk, i.e. the simplicial complex with 3 vertices, 3 line segments joining them (1-simplices) and also the big 2-simplex, so that our triangle is not hollow. (no holes, not the circle)

Let’s call that \( K^0 \). And now let’s call the n-th barycentric subdivision \( K^n \). Let \( s_0(n) \) be the number of vertices of \( K^n \), \( s_1(n) \) be the number of 1-simplices of \( K^n \) and \( s_2(n) \) the number of 2-simplices.

Note that for our \( K^0 \) we have \( s_0(0) = 3, s_1(0) = 3, \) and \( s_2(0) = 1. \)

End definitions

1. For starters, draw pictures of the first and second barycentric subdivisions. To check your work, we have \( s_0(1) = 7, s_1(1) = 12, s_2(1) = 6, s_0(2) = 25, s_1(2) = 60, \) and \( s_2(2) = 36. \)

2. Find recursive formulas for \( s_0(n), s_1(n), \) and \( s_2(n). \) That is, write them all in terms of \( s_0(n - 1), s_1(n - 1), \) and \( s_2(n - 1). \)

3. Prove that \( s_2(n) - s_1(n) + s_0(n) = 1. \)

4. Find closed formulas for \( s_0(n), s_1(n), \) and \( s_2(n). \) That is, we don’t want a recursive formula, but something you just plug in \( n \) to and get your answer, e.g. \( s_2(n) = 6^n \) (in case you hadn’t already noticed).