Wei-Liang Chow

W. Stephen Wilson

I was hired by Johns Hopkins University the year Professor Chow retired, 1977. Of course, names like Chow's helped to make Hopkins an attractive place to be. My contact with him was at department parties over the next 15 years. It was always a pleasure to see him. He was fascinating to listen to about his personal history. He was a bouncy, cheery, fun person to be around. I am happy to have known him and honored to be able to help organize this memorial to him.

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Wei-Liang Chow 1911-1995

S. S. Chern

After a long illness Wei-Liang died on August 10, 1995. He and I first met in Hamburg, Germany in October 1934, when I had just come from China as an entering student, while he was on his way from Göttingen to Leipzig in order to work with van der Waerden.

Wei-Liang came from a Mandarin family, one whose leading members realized the importance of the westernization of China. With their resources and enlightened view the family produced during the turn of the century several leaders of Chinese society in different areas. Perhaps as a result of the family situation Wei-Liang did not attend a Chinese school. However, through private tutoring he is quite familiar with the Chinese language and Chinese history. His family position allowed him to go through college in the U.S., receiving his B.A. from the University of Chicago in 1931. His

Chicago years gradually focused him to mathematics and in 1932 he went to Göttingen, then one of the greatest mathematical centers in the world. Unfortunately political events in Germany during this period made his stay in Göttingen undesirable and he decided to go to Leipzig to work with van der Waerden.

It was at this juncture that we met in Hamburg. The decline of Göttingen had the result of elevating Hamburg to a leading mathematical center in Germany. Her leading attraction was Emil Artin, the young professor who gave excellent lectures and whose interest extended over all areas of mathematics. Although Wei-Liang was a Leipzig student, the German university system allowed him to live in Hamburg. Besides the contacts with Artin, he had a more important objective, which was to win the love of a young lady, Margot Victor. They were married in 1936, and I was fortunate to be present at the wedding.

After their marriage Wei-Liang returned to China and became a professor of mathematics at the Central University in Nanking, then the Chinese capital. The next years China was at war, with the coastal provinces occupied by the Japanese.

We next saw each other in 1946 in Shanghai after the war ended. In a decade of war years Wei-Liang had practically stopped his mathematical activities, and the question was whether it was advisable or even possible for him to come back to mathematics.

His return to mathematics was most successful; I would consider it a miracle. He began by spending the years 1947-49 at the Institute for Advanced Study, after which he accepted a position at Johns Hopkins University from which he retired in 1977. At Johns Hopkins he served as chairman for more than ten years. He was also responsible for the *American Journal of Mathematics*, a Hopkins publication and the oldest American mathematical journal.

Wei-Liang was an original and versatile mathematician, although his major field was algebraic geometry. He made several fundamental contributions to mathematics as follows:

1. A fundamental issue in algebraic geometry is intersection theory. The Chow ring has many advantages and is widely used.

2. The Chow associated forms give a description of the moduli space of the algebraic varieties in projective space. It gives a beautiful solution of an important problem.

3. His theorem that a compact analytic variety in a projective space is algebraic is justly famous. The theorem shows the close analogy between algebraic geometry and algebraic number theory.

4. Generalizing a result of Caratheodory on thermodynamics, he formulated a theorem on accessibility of differential systems. The theorem plays a fundamental role in control theory.

5. A lesser known paper of his on homogeneous spaces gives a beautiful treatment of the geometry known as the projective geometry of matrices and treated by elaborate calculations. His discussions are valid in a more general context.

Chow led a simple and secluded life with complete devotion to mathematics and some other intellectual activities, including philately. He was an authority on Chinese stamps and published a book on them. Margot and Wei-Liang have three daughters and a happy family life.

Undoubtedly he is an algebraic geometer of the first caliber. I once nominated him for membership in the National Academy of Sciences with the support of Zariski. Unfortunately it did not meet with success and I think it was a loss to the Academy.

Wei-Liang Chow

Shreeram S. Abhyankar

Eddie, as Chow was fondly called by family and friends, was such a talkative friendly soul that, at a recent get-together, when I classified people as my students and teachers, inspite of the twenty year difference in our ages, I counted him amongst my chums. Indeed, Eddie's loving wife Margot used to comment that when I visited them, his voice rose a notch after all-night chats with me. Those days, we both used to be night workers, who worked until four at dawn, and got up around noon. I remember Eddie telling me that he accepted the chairmanship at Hopkins with the proviso that he may come to the department only in the afternoon, and that he would carry on the departmental business mostly on the phone rather than by letters. We may wonder how he would have reacted to the modern electronic age of e-mail.

My fond memories of the Chow family go back to the days when in the summer of 1957 we stayed in nearby cottages on China Lake in Maine. During the day I used to do Yoga Meditation, and during the night Eddie and I used to have long discussions comparing Chinese and Indian cultures and whether it is better to do Yoga or Mathematics. Our discussions came to a sudden halt when I broke my bones in a car accident and was hospitalized for a couple of months. Not wanting to gain weight, I had requested the nurses to put me on a diet. But then during the night I used to feast on the delicious rum cake which Margot Chow would bake for me.

Chow's name has become a household word in mathematics because of Chow Coordinates and Chow's Theorem on analytic sets in projective spaces.

Chow coordinates are a natural generalization of Grassmann coordinates. An $(m+1)$-dimensional subspace W of an $(n+1)$-dimensional vector space V can be specified by an $(m+1)$ by $(n+1)$ matrix whose rows are the coordinates of a basis of W. The set of all $(m+1)$ by $(m+1)$ minors of this matrix are the Grassmann coordinates of W. There are $\binom{n+1}{m+1}$ of these. They are determined by W up to a constant of proportionality. Consider the n-dimensional projective space $\mathcal{P}(V)$ associated with V, whose points are the 1-dimensional subspaces of V. As W varies over the set of all $(m+1)$-dimensional subspaces of V, the associated projective space $\mathcal{P}(W)$ varies over the set $G_{n,m}$ of all m-dimensional (projective) subspaces of $P^n = \mathcal{P}(V)$. The Grassmann coordinates give us an embedding of the Grassmann variety $G_{m,n}$ into the projective space P^N with $N = \binom{n+1}{m+1} - 1$. It can be shown that $G_{n,m}$ is the intersection of a certain number of hyperquadrics in P^N. Systems of m-dimensional subspaces of P^n correspond to subvarieties of $G_{n,m}$. For example a one-parameter family of planes in P^n corresponds to a curve in $G_{n,2}$. Subvarieties of Grassmann varieties were studied by Grassmann and Plücker in the last century.

If, more generally, we want to study systems of m-dimensional varieties of degree d in P^n then we must replace Grassmann coordinates by Chow coordinates. To define these we intersect a given m-dimensional variety Z of degree d by a generic $(n-m)$-dimensional subspace U of P^n. The coordinates of the d points of intersection are algebraic functions of the Grassmann coordinates of U. By taking a symmetric function of the algebraic functions we get a homogeneous polynomial which is the Chow form of Z. Coefficients of the Chow form are the Chow coordinates. In this manner we embed the set $C_{n,m,d}$ of all m-dimensional varieties of degree d in P^n into the projective space P^M of certain dimension M. In his fundamental 1937 paper (vol. 113 of Math. Annalen) Chow proved that the Chow variety $C_{n,m,d}$ is indeed an algebraic variety in P^M. Although this was a joint paper of Chow and van der Waerden, it was explicitly mentioned that the material dealing with Chow forms was due to Chow. When, during 1952-53, I was a graduate student of Zariski, I read the theory of Chow coordinates in Zariski's forthcoming book on Algebraic Geometry (= the unborn parent cited in the Introduction of Zariski-Samuel's Commutative Algebra). Since Chow forms first appeared in his joint paper with van der Waerden, they are sometimes called Chow-van-der-Waerden forms. In their 3 volume book Methods of Algebraic Geometry, Hodge and Pedoe say that the idea of Chow forms goes back to Cayley and so they call them Cayley forms. Chow himself called them Canonical forms and joked that all the three alternatives may be abbreviated as C-forms. At any rate, Chow forms have been a very versatile tool in many aspects of algebraic geometry.

Turning to Chow's Theorem on analytic sets in projective spaces, assuming the ground field to be the field of complex numbers, it says that every such set is algebraic; see Chow's 1949 paper in volume 71 of American Journal of Mathematics. This is a marvelous generalization of Liouville's Theorem which itself generalizes the so-called Fundamental Theorem of Algebra. Chow's original proof was based on ideas from algebraic number theory.

Chow also made two important contributions to algebraic geometry over a ground ring. The first of this was an extension of Bertini's Theorem to a local domain; see his 1958 paper in volume 44 of Proc. of NAS; this was done in response to my request as I needed it in my work on local fundamental groups of algebraic varieties. The second was Chow's simplified version of Zariski's Connectedness Theorem as given in his 1959 paper in volume 81 of American Journal of Mathematics.

On the side of pure geometry, to quote from page 212 of Artin's 1957 book on Geometric Algebra, "Chow's 1949 paper (volume 50 of Annals of Mathematics) On the Geometry of Algebraic Homogeneous Spaces is one of the most fascinating developments in projective geometry."

Under Chow's editorship, from 1953 to 1977, the American Journal of Mathematics was a very prosperous place for papers in Algebraic Geometry. During the said period, I myself published 19 papers in that journal.

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Comments on Chow's Works

Serge Lang

Van der Waerden's pre-war series of articles began an algebraization of Italian algebraic geometry. I was born into algebraic geometry in the immediate post war period. This period was mostly characterized by the work of Chevalley, Chow, Weil (starting with his *Foundations* and his books on correspondences and abelian varieties), and Zariski. In the fifties, there was a constant exchange of manuscripts among the main contributors of that period. I shall describe briefly some of Chow's contributions. I'll comment here mostly on some of Chow's works in algebraic geometry, which I know best.

§1. Chow coordinates

One of Chow's most influential works was also his first, namely the construction of the Chow form, in a paper written jointly with van der Waerden [ChW 37a]. To each projective variety, Chow saw how to associate a homogeneous polynomial in such a way that the association extends to a homomorphism from the additive monoid of effective cycles in projective space to the multiplicative monoid of homogeneous polynomials, and the association is compatible with the Zariski topology. In other words, if one cycle is a specialization of another, then the associated Chow form is also a specialization. Thus varieties of given degree in a given projective space decompose into a finite number of algebraic families, called Chow families. The coefficients of the Chow form are called the Chow coordinates of the cycle, or of the variety. Two decades later, he noted that the Chow coordinates can be used to generate the smallest field of definition of a divisor [Ch 50a]. He also applied the Chow form to a study of algebraic families when he gives a criterion for local analytic equivalence [Ch 50b]. He was to use them all his life, in various contexts dealing with algebraic families.

In Grothendieck's development of algebraic geometry, Chow coordinates were bypassed by Grothendieck's construction of Hilbert schemes, whereby two schemes are in the same family whenever they have the same Hilbert polynomial. The Hilbert schemes can be used more advantageously than the Chow families in some cases. However, as frequently happens in mathematics, neither is a substitute for the other in all cases. In recent times, say during the last decade, Chow forms and coordinates have made a reappearance due to a renewed emphasis on explicit constructions needed to make theorems effective (rather than having non-effective existence proofs, say), and for computational aspects of algebraic geometry whereby one wants not only theoretical effectiveness but good bounds for solutions of algebraic geometric problems as functions of bounds on the data. Projective constructions such as Chow's are very well suited for such purposes. Thus Chow coordinates reappeared both in general algebraic geometry, and also in Arakelov theory and in diophantine applications. The Chow coordinates can be used for example to define the height of a variety, and to compare it to other heights constructed by more intrinsic, non-projective methods as in [Ph 91], [Ph 94], [Ph 95]. They were used further in Arakelov theory by Bin Wang [Wa 96].

Chow coordinates were also used to prove a conjecture of Lie on a converse to Abel's theorem. See the papers by Wirtinger [Wi 38] and Chern [Che 83].

§2. Abelian varieties and group varieties

(a) **Projective construction of the Jacobian variety.** In the fifties, Chow contributed in a major way to the general algebraic theory of abelian varieties due to Weil (who algebraicized the transcendental arguments of the Italian school, especially Castelnuovo). For one thing, Chow gave a construction of the Jacobian variety by projective methods, giving the projective embedding directly and also effectively [Ch 54]. The construction also shows that when a curve moves in an algebraic family, then the Jacobian also moves along in a corresponding family.

(b) **The Picard variety.** Chow complemented Igusa's transcendental construction of the Picard variety by showing how this variety behaves well in algebraic systems, using his "associated form" [Ch 52b]. He announced an algebraic construction of the Picard variety in a "forthcoming paper." Indeed, such a paper circulated as an unpublished manuscript a few years later [Ch 55c], but was never published as far as I know.

(c) **Fixed part of an algebraic system.** Chow also developed a theory of algebraic systems of abelian varieties, defining the fixed part of such systems, i.e. that part which does not depend genuinely on the parameters [Ch 55a,b]. His notion of fixed part was used by others in an essential way, e.g. by Lang–Néron, who proved that for an abelian variety A defined over a function field K, the group of rational points of A in K modulo the group of points of the fixed part is finitely generated [LaN 59]. This is a relative version of the Mordell–Weil theorem.

(d) **Field of definition.** Chow gave conditions under which an abelian variety defined over an extension of a field k can actually be defined over k itself [Ch 55a,b]. Chow's idea was extended by Lang [La 55] to give such a criterion for all varieties, not just abelian varieties, and Weil reformulated the criterion in terms of cohomology (splitting a cocycle) [We 56].

§3. Homogeneous spaces

(a) **Projective embedding of homogeneous spaces.** Chow extended the Lefschetz–Weil proof of the projective embedding of abelian varieties to the case of homogeneous spaces over arbitrary group varieties, which may not be complete [Ch 57a]. Chow's proof has been overlooked in recent years, even though interest in projective constructions has been reawakened, but I expect Chow's proof to make it back to the front burner soon, just like his other contributions.

(b) **Algebraic properties.** Chow's paper [Ch 49b] dealt with the geometry of homogeneous spaces. The main aim of this paper is to characterize the group by geometric properties. The latter could refer to the lines in a space, as in projective geometry, or to certain kinds of matrices, such as symmetric matrices. For instance, a typical theorem says: Any bijective adjacence preserving transformation of the space of a polar system with itself is due to a transformation of the basic group, provided that the order of the space is greater than 1. Birational geometry is considered in this context.

§4. The Chow ring

In topology, intersection theory holds for the homology ring. In 1956, Chow defined rational equivalence between cycles on an algebraic variety, he defined the intersection product for such classes, and thus obtained the Chow ring [Ch 56a], which proved to be just as fundamental in algebraic geometry as its topological counterpart.

§5. Algebraic geometry over rings

In the late fifties began the extension of algebraic geometry over fields to algebraic geometry over rings of various type, partly to deal with algebraic or analytic families, but partly because of the motivation from number theory, where one deals with local Dedekind rings, p-adic rings, and more generally complete Noetherian local rings. Chow contributed to this extension in several ways. Of course, in the sixties Grothendieck vastly and systematically went much further in this direction, but it is often forgotten that the process had begun earlier. I shall mention here some of Chow's contributions in this direction.

(a) **Connectedness theorem.** In 1951 Zariski had proved a general connectedness theorem for specializations of connected algebraic sets. Zariski based his proof on an algebraic theory of holomorphic functions which he developed for this purpose. In [Ch 57b] and [Ch 59], Chow gave a proof of a generalization over arbitrary complete Noetherian local domains, based on much simpler techniques of algebraic geometry, especially the Chow form.

(b) **Uniqueness of the integral model of a curve.** The paper [ChL 57c] proved the uniqueness of the model of a curve of genus ≥ 1 and an abelian variety over a discrete valuation ring, in the case of non-degenerate reduction.

(c) **Cohomology.** Invoking the theory of deformations of complex analytic structures by Kodaira–Spencer, the connectedness theorem, and Igusa's work on moduli spaces of elliptic curves, Chow and Igusa proved the upper semicontinuity of the cohomology over a broad class of Noetherian local domains [ChI 58c]. Semicontinuity was proved subsequently in the complex analytic case by Grauert, and by Grothendieck in more general algebraic settings. However, Chow's and Igusa's contribution did not get the credit they deserved. Cf. [Ha 77], Chapter 3, §12, and the bibliographical references given there, referring to work in the sixties, but not to Chow–Igusa.

(d) **Bertini's theorem.** During that same period in the late fifties, Chow extended Bertini's theorem to local domains [Ch 58b].

(e) **Unmixedness theorem.** A homogeneous ideal defining a projective variety is said to be unmixed if it has no embedded prime divisors. Chow proved that the Segre product of two unmixed ideals is also unmixed, under fairly general conditions, in a ring setting [Ch 64].

§6. Algebraicity of analytic objects

Chow was concerned over many years with the algebraicity of certain complex analytic objects. We mention two important instances.

(a) **Meromorphic mappings and formal functions.** In 1949, Chow proved the fundamental fact, very frequently used from then on, that a complex analytic subvariety of projective space is actually algebraic [Ch 49a]. Twenty years later, he came back to similar questions, and proved in the context of homogeneous varieties that a meromorphic map is algebraic [Ch 69]. Remarkably, and wonderfully, almost twenty years after that, he came back once more to the subject and completed it in an important point [Ch

86]. I quote from the introduction to this paper, which shows how Chow was still lively mathematically: "Let X be a homogeneous algebraic variety on which a group G acts, and let Z be a subvariety of positive dimension. Assume that Z generates X [*in a sense which Chow makes precise*]...One asks whether a formal rational function on X along Z is the restriction along Z of an algebraic function (or even a rational function) on X. In a paper [Ch 69] some years ago, the author gave an affirmative answer to this question, under the assumption that the subvariety Z is complete, but only for the complex-analytic case with the formal function replaced by the usual analytic function defined in a neighborhood of Z. The question remains whether the result holds also for the formal functions in the abstract case over any ground field. We had then some thoughts on this question, but we did not pursue them any further as we did not see a way to reach the desire conclusion at the time. In a recent paper [3], Faltings raised this same question and gave a partial answer to it in a slightly different formulation. This result of Faltings led us to reconsider this question again, and this time we are more fortunate. In fact, we have been able not only to solve the problem, but also to do it by using essentially the same method we used in our original paper."

(b) **Analytic surfaces.** In a paper with Kodaira, it was proved that a Kähler surface with two algebraically independent meromorphic functions is a non-singular algebraic surface [ChK 52c].

§7. Other works in algebraic geometry

Chow's papers in algebraic geometry include a number of others, which, as I already asserted, I am less well acquainted with, and won't comment upon, such as his paper on the braid group [Ch 48], on the fundamental group of a variety [Ch 52d], on rational dissections [Ch 56b], and on real traces of varieties [Ch 63].

§8. PDE

Chow's very early paper on systems of linear partial differential equations of first order [Ch 39c] gives a generalization of a theorem of Caratheodory on the foundations of thermodynamics. This paper had effects not well known to the present generation of mathematicians, including me. It was only just now brought to my attention. An anonymous colleague wrote to the editor of the present collection of articles on Chow's work: "This paper essentially asserts the identity of the integral submanifold of a set of vector fields and the integral submanifold of the Lie algebra generated by the set of vector fields. This is widely known as "Chow's theorem" in nonlinear control theory, and is the basis for the study of the controllability problem in nonlinear systems. Controllability refers to the existence of an input signal that drives the state of a system from a given initial state to a desired terminal state. A more detailed exposition of the role of Chow's theorem, with several references, is provided in the survey paper [Br 76]."

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In Memory of Professor Wel-Liang Chow

Jun-ichi Igusa

When I was a student at Tokyo University, I saw W.-L. Chow's name in van der Waerden's introductory book to algebraic geometry. Although it was clearly stated in their joint paper quoted in the book, I learned of Chow's contribution later in Kyoto through the following statement by van der Waerden in his article of 1948:

> As the eminent Chinese mathematician Wei-Liang Chow came to Leibzig, I discussed with him the following problem: How can one represent any algebraic manifold by one single algebraic equation?

The main theorem of Cayley forms, proved by Chow, asserts the existence of a set of algebraic relations among the coefficients of a form $f(u, u^{(1)}, \ldots, u^{(d)})$, necessary and sufficient in order that it be the Cayley form of a manifold M. In this situation, I, Matsusaka and students of algebraic geometry in Kyoto University started using "Chow forms" (instead of Cayley forms, etc.), "Chow points," and "Chow varieties" possibly for the first time in the world.

I met Professor Chow three times in 1954 at Princeton, Boulder and Cambridge. My close association with him started in 1955 when he became the chairman of the Mathematics Department at Hopkins and I joined him with Sampson that year. Washnitzer came to Hopkins one year earlier and Dwork one or two years later.

Chow was 12 years older than Dwork and starting with him we are 1 year younger in alphabetical order. In 1959 Abhyankar also joined to this by then well-established group of algebraic geometers with Chow as its leader. All young members had been either at Harvard or MIT and had known each other since that time. We were very close both mathematically and socially. Professors Weil and Zariski visited Hopkins regularly in those days. Among algebraic geometers of our age I remember Lang very well as a regular visitor. Those were some of the most fondly remembered years of my life and I would think that a similar feeling might be shared by all of us.

It is a historical fact that this school of algebraic geometry was "created" by Professor Chow. He was very open and free in expressing his ideas while they were still in a nebulous state. We often saw how many of those ideas evolved into beautiful theorems. We were attracted by his openness and impressed by his superb geometric intuition. Furthermore, as Washnitzer used to mention, Chow founded this school without causing any financial strain to the University. I might add that Professor Chow's wife, Margot, also contributed to our social closeness at that time. We became almost like relatives—some of us even spent summers together with the Chows at China Lake, Maine.

In the spring of 1962 Professor Chow succeeded in persuading Professor Kodaira to come to Hopkins. Kodaira already spent the year 1950-51 at Hopkins to work with Chow. Also almost all of us spent a month at Boulder with Kodaira in 1954. With Kodaira's arrival in September, the school of algebraic geometry at Hopkins reached its peak, and it was compared favorably by Grothendieck and others with the Harvard school of algebraic geometry with Zariski as its leader. As it happens at almost all phases of human history, once a peak is reached by definition, a decline becomes inevitable. The primary reason for the decline of Hopkins school of algebraic geometry was the lack of funds to sustain it. Young members of the school who came mostly as assistant professors, were no longer young. Furthermore by the excellent pieces of work they produced, most of them started getting offers from other leading universities which Hopkins found very difficult to match. Chow, even with his unusual administrative talent, had to watch the breakup of the school he created knowing that the leaving members who were so dear to him were unreplaceable. In the year 1965, when Kodaira left Hopkins, Chow stepped down from his chairmanship and was succeeded by Professor Hartman.

The Hopkins Mathematics Department entered a new era with Hartman as its chairman. The country itself was changing at that time, and Chow was supportive of Johnson's Great Society Program. However both Professor & Mrs. Chow held a traditional view on family. I remember the story which they told us much later about their youngest daughter: She and her husband graduated from Harvard Law School, but after having children she quit being a full-time lawyer because she felt that the keeping of her house was her primary responsibility. They supported her view wholeheartedly.

Professor Chow retired from Hopkins in 1977. We continued to meet socially at large parties with other departmental members and also at several of their favorite restaurants two or three times a year. At one such evening he showed us his stamp collection. Chow was a stamp collector well known not only among mathematicians but also among professional stamp collectors. He said that the book he showed us was not his best which were kept in a safe at their bank. To us, however, the book with so precisely and beautifully arranged stamps was an object of art—it was simply fantastic. I might also mention another story. This was in the late 50's and we were living next to each other on a street called Midwood Avenue. He said that he was going to have a house built—he bought a lot and was just trying to find a builder. He showed me the blueprint of the house which he himself drew up. It turned out, however, that he was unable to find a builder because no one wanted to sign a contract with a customer who knew, at least theoretically, far more about home design, construction, and materials than any builder.

In the later years of our meetings, Professor & Mrs. Chow often mentioned the time when they were in China. After their marriage in Hamburg in July of 1936, they left Nazi Germany for China and Chow started teaching at the Central University in Nanjing in September of that year. However only one year later they found that China was no better than Germany. Imperial Japan enlarged a small fight on July 7, 1937 at the Marco Polo Bridge near Beijing to a systematic invasion of China. On August 13 a skirmish occurred in Shanghai and on December 13 the "Rape of Nanjing" started. Fortunately they escaped Nanjing in September of that year to Chow's birth place, Shanghai. Being an international city, they felt safer there. They told us, however, that Shanghai at that time was quite similar to the Shanghai described in S. Spielberg's movie, "Empire of the Sun. "In the first two to three years in China, Chow was still able to communicate with mathematicians in Europe, especially with van der Waerden. However during the remaining 8 years before he came to the United States the situation became so bad that he was unable to continue his mathematics. He told us more than once that it was Professor Chern who encouraged and helped him to come back to mathematics. Chow came to the Institute for Advanced Study in Princeton in March of 1947 and to Hopkins in the fall of 1948. He went on to say that without Chern's friendship that might not have taken place.

The last time we met Professor & Mrs. Chow socially was in August of 1994 and the last time we saw Chow was in June of 1995. He died peacefully shortly after that at 2 a.m. on August 10. The memory of Professor Chow by those who knew him and the mathematics he created will live on. I feel fortunate to have known a great man like Professor Wei-Liang Chow for 40 years and am grateful for his kindness during those years.