1. **Class groups of affine varieties.** Let $X$ be a non-singular affine variety. Prove that $\text{Cl}(X) = 0$ if and only if the coordinate ring $k[X]$ is a UFD.

2. Let $X$ be a non-singular variety. Prove that the projection $\pi : X \times \mathbb{A}^1 \to X$ induces a surjective homomorphism $\pi^* : \text{Cl}(X) \to \text{Cl}(X \times \mathbb{A}^1)$.

3. Let $X$ be a non-singular variety. Use the previous problem to prove that $\text{Cl}(X \times \mathbb{A}^1)$ is isomorphic to $\text{Cl}(X)$.

4. **Rational points on elliptic curves.** Let $E : y^2 = x^3 + ax + b$ be a non-singular cubic curve with $a$ and $b$ in a number field $k$. Prove that the $k$-points $E(k)$ form a subgroup of the complex torus $E(\mathbb{C})$. (Recall that the curve $E$ over the field of complex numbers is a complex torus.)

5. Show that the equation $y^2 = x^3 - 2$ has infinitely many rational solutions.