Problem 0.1. Recall that the twisted cubic curve $C$ is the image of the map $\nu_3 : \mathbb{P}^1 \rightarrow \mathbb{P}^3$ given by $[x_0 : x_1] \mapsto [x_0^3 : x_0^2 x_1 : x_0 x_1^2 : x_1^3]$.

(1) Show that the homogeneous ideal of the image $C$ is generated by $Q_1 : z_0 z_2 = z_1^2$, $Q_2 : z_1 z_3 = z_2^2$, $Q_3 : z_0 z_3 = z_1 z_2$.

(2) Show that an alternative way to describe the twisted cubic is as the rank one locus of the matrix

\[
\begin{bmatrix}
z_0 & z_1 & z_2 \\
z_1 & z_2 & z_3
\end{bmatrix}
\]

(3) Show that $Q_i \cap Q_j$ for $i \neq j$ is $C$ union a line.

Problem 0.2. Let $X_1$ and $X_2$ be algebraic sets in $\mathbb{P}^n$. They are said to be projectively equivalent if there is a linear transformation $\Phi \in \text{PGL}(n+1, k)$ takes one to the other: $\Phi(X_1) = X_2$ (where $k$ is always an algebraically closed field).

(1) Show that any two ordered sets of $n + 2$ points in general position in $\mathbb{P}^n$ are projectively equivalent. General position: no three collinear, no six coplanar, etc.

(2) Show that two sets of fours points in $\mathbb{P}^1$ are projectively equivalent if and only if there cross ratios are equal.

Problem 0.3. (1) $\dim \mathbb{P}^n = n$.

(2) A projective variety $X \subseteq \mathbb{P}^n$ has dimension $n - 1$ if any only if it is the zero set of a single irreducible homogeneous polynomial $f$ of positive degree. $X$ is called a hypersurface in $\mathbb{P}^n$.

Problem 0.4. Linear varieties in $\mathbb{P}^n$. A hypersurface defined by a linear polynomial is called a hyperplane.

(1) Show that the following two conditions are equivalent for a variety $X$ in $\mathbb{P}^n$:
   (a) $I(X)$ can be generated by linear polynomials.
   (b) $X$ can be written as an intersection of hyperplanes.

   In this case we say that $X$ is a linear variety in $\mathbb{P}^n$.

(2) If $X$ is a linear variety of dimension $r$ in $\mathbb{P}^n$, show that $I(X)$ is minimally generated by $n - r$ linear polynomials.

(3) Let $Y_1$ and $Y_2$ be linear varieties in $\mathbb{P}^n$ with $\dim Y_1 = r$, $\dim Y_2 = s$. If $r + s - n \geq 0$, then $Y_1 \cap Y_2 \neq \emptyset$. Furthermore, if $Y_1 \cap Y_2 \neq \emptyset$, then $Y_1 \cap Y_2$ is a linear variety of dimension at least $r + s - n$.

Problem 0.5. (1) Show that the Segre image of $\mathbb{P}^1 \times \mathbb{P}^1$ is a quadric surface in $\mathbb{P}^3$.

(2) Let $L, M,$ and $N$ be three pairwise skew lines in $\mathbb{P}^3$. Show that unions of all the lines in $\mathbb{P}^3$ intersecting $L, M, N$ is isomorphic to the Segre image of $\mathbb{P}^1 \times \mathbb{P}^1$. 

1