Math 211, Spring 2018: Homework Set 8

1. Section 6.1: 2, 4, 16, 23, 31, 39
2. Section 6.2: 4, 8, 9, 17, 21(polar coordinates)
3. **Winding number**. This question will lead you to investigate line integrals of irrotational vector fields. Suppose $\mathbf{F}$ is a continuously differentiable vector field in a multiply-connected – i.e., not simply connected – region D of the xy-plane, and suppose $\text{curl}(\mathbf{F}) = 0$. Let $C = \partial D$ be the boundary curve of D. Suppose C does not pass through the origin of the plane.

   (a) Consider the example of the vector field representing the electromagnetic field of a wire along the z-axis carrying a constant current:

   $$\mathbf{F} = \frac{-y\mathbf{i} + x\mathbf{j}}{r^2};$$

   (can easily see that $\text{curl}(\mathbf{F}) = 0$.) Note that this field is not defined at the origin of the plane. Using change of variable formula to convert to the polar coordinates and express the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C \frac{-ydx + xdy}{r^2}$ as an integral with respect to θ.

   (b) Show that the integral from part (a) is equal to $2\pi n$ for some integer n. This number n is called the *winding number* of C around the origin.

   (c) Show that the number n also equals the number of times C crosses the positive x-axis, counting $+1$ if it crosses from below to above and -1 if it crosses from above to below.