1. Evaluate the following:

(a) \[ \int \arcsin x \, dx \]

(b) \[ \int \frac{e^{2x} \, dx}{e^{2x} - 1} \]

(c) \[ \frac{d}{dx} \int_{\arcsin x}^{\pi/2} \ln(\sin t) \, dt \]
2. The volume \( V \) (in cubic inches) and pressure \( p \) (in pounds per square inch) of the air inside a balloon satisfy the equation \( pV = 1000 \). At what rate is the volume of the balloon changing if the pressure is currently 100 \( lb/in^2 \) and is dropping at the rate of 2 \( lb/in^2 \) per second?
3. Solve the initial value problem:

\[ \frac{dy}{dx} = \frac{\sqrt{x}}{1 + x}, \quad y = 2 \text{ when } x = 1. \]
4. (a) Find the first (linear) and second (quadratic) Taylor polynomials for the function $f(x) = \sqrt{x}$ centered about the point 1.

(b) Use part (a) to find the first- and second-order approximations to the exact value of $\sqrt{101}$. [Hint: $\sqrt{101} = 10\sqrt{1.01}$]
5. Decide if the improper integral
\[ \int_{1}^{\infty} \frac{x \, dx}{x^4 + 1} \]
converges or diverges. Justify your answer.
6. A rectangle with sides parallel to the coordinate axes has one vertex at the origin, one on the positive $x$-axis, one on the positive $y$-axis, and its fourth vertex on the line $y = 100 - 2x$ (see the figure). What is the maximum possible area of such rectangle? What are its dimensions?

Figure 1: The rectangle inscribed under $y = 100 - 2x$. 

![The rectangle inscribed under y = 100 - 2x.](image-url)
7. A spherical container of radius 10 feet is partly filled with water (see figure). The depth of the water remaining in the container is 5 feet. Would you intuitively expect that the container is exactly one-quarter full, or less than one-quarter full, or more? Find the exact volume of the water inside (for reference, the capacity of the full container is \( \frac{4000}{3} \pi \) cubic feet.)

[Hint: Find the volume of the solid of revolution obtained by revolving the shaded circular wedge of the semi-circle \( y = \sqrt{100 - x^2} \) around the \( x \)-axis.]

Figure 2: Cross-section of the water container.
8. The graph of \((x^2 + y^2 + y)^2 = x^2 + y^2\) is a cardioid ("heart-shaped").

Find the equation of the tangent line to the cardioid passing through the point (1,0). (Hint: use implicit differentiation.)

\[\text{Figure 3: The cardioid } (x^2 + y^2 + y)^2 = x^2 + y^2.\]