201 Linear Algebra, Practice Midterm2

Duration: 50 mins

1. \[ A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \]

Find the matrix of the transformation \( T(\vec{x}) = A\vec{x} \) with respect to the basis \( \{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \} \).

2. \( T(M) = M \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \) defines a linear transformation on the space of \( 2 \times 2 \) matrices; \( T : \mathbb{R}^{2 \times 2} \to \mathbb{R}^{2 \times 2} \).
   
   (a) Write down the matrix for this transformation in terms of the standard basis for \( \mathbb{R}^{2 \times 2} \)
   
   (b) Find the Kernel and Image of \( T \).
   
   (c) Find the matrix of \( T \) with respect to the basis \( B = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \right\} \).
   
   (d) How is the matrix in part (c) related to the matrix in part (a)?

3. Find an orthonormal basis for the subspace of \( \mathbb{R}^4 \) consisting of all those vectors that are perpendicular to \( \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \).

4. Find the least-squares solution \( \vec{x}^* \) of the system \( \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} \).

5. TRUE or FALSE. Justify your answer.
   
   (a) All linear transformations from \( P_3 \) to \( \mathbb{R}^{2 \times 2} \) are isomorphisms
   
   (b) If the matrix of a linear transformation \( T : V \to V \) with respect to some basis is invertible, then \( T \) is invertible.
   
   (c) If the \( 2 \times 2 \) matrix \( R \) represents the reflection about a line in \( \mathbb{R}^2 \), then there is an invertible \( 2 \times 2 \) matrix \( S \) such that \( R = S^{-1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} S \).
   
   (d) If a matrix \( A \) is similar to \( B \), and \( A \) is orthogonal, then so is \( B \).
   
   (e) If the matrix \( A \) is orthogonal then \( A^3 \) is orthogonal.