PRACTICE PROBLEMS: SERIES

1. Assume $a_n$ are such that $\sum_{n=1}^{\infty} 2^n a_n$ is convergent.
   a) True or false: $\sum_{n=1}^{\infty} a_n$ is convergent.
   b) True or false: $\sum_{n=1}^{\infty} (-1)^n 2^n a_n$ is convergent.
   c) True or false: $\sum_{n=1}^{\infty} n a_n$ is convergent.

2. Determine the radius and domain of convergence for $\sum_{n=0}^{\infty} 2^n x^{n^2}$.

3. a) Determine the radius and domain of convergence for $\sum_{n=0}^{\infty} (-1)^n x^{2n+1} (2n+1)!$.
   b) Let $f(x)$ the sum of the series from a). Prove that $f$ satisfies the equation $f''(x) = -f(x)$ on the domain of convergence.

4*. Find the sum of the series $\sum_{n=0}^{\infty} (-1)^n$.

5*. Prove Abel’s theorem.