202 Practice Final

1. Calculate $\nabla \times \mathbf{V}$, where $\mathbf{V}(x,y,z) = \mathbf{A} \times \mathbf{r}$, with $\mathbf{r}$ equal to the radial vector $(x,y,z)$, and $\mathbf{A} = (a,b,c)$ a \textbf{constant} vector.

2. Find a function $f(x,y)$ with
   $$\nabla f = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right).$$

3a. Calculate the integral
   $$\int \int_H \mathbf{r} \cdot d\mathbf{S}$$
   with $\mathbf{r}$ as above, where $H$ is the spiral ramp defined by the parametrization
   $$\mathbf{R}(u,v) = (u \cos v, u \sin v, v),$$
   with $0 \leq u \leq a$, $0 \leq v \leq 2\pi$.

3b. Calculate the integral
   $$\int_{\partial H} \mathbf{r} \cdot d\mathbf{s}.$$

4. Rotating a parametrized curve $\mathbf{c}(t) = (c_1(t), c_2(t))$, where $a \leq t \leq b$, around the $x$-axis defines a surface of revolution parametrized by
   $$\mathbf{R}(u,v) = (c_1(u), c_2(u) \cos v, c_2(u) \sin v),$$
   where $a \leq u \leq b$, $0 \leq v \leq 2\pi$. Calculate the \textbf{normalized} normal vector defined by this parametrization, when the original curve is the cycloid defined by
   $$\mathbf{c}(t) = (t - \sin t, 1 - \cos t), 0 \leq t \leq 2\pi.$$
   [Answer: The \textit{un}normalized normal vector is
   $$(1 - \cos u)(\sin u, -(1 - \cos u) \cos v, -(1 - \cos u) \sin v),$$
   which has length squared equal to $2(1-\cos u)^3$; so the final answer is a little messy.]

5. Calculate the volume of the ellipsoid
   $$\mathbf{E} = \{(x,y,z) \in \mathbb{R}^3 \mid \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 \leq 1\}$$
   by using the parametrization
   $$\mathbf{T}(u,v,w) = (aw \cos u \cos v, bw \sin u \cos v, cw \sin v),$$
   where $0 \leq u \leq 2\pi$, $-\frac{1}{2}\pi \leq v \leq \frac{1}{2}\pi$, $0 \leq w \leq 1$.
   [Answer: This boils down to calculating the triple integral of $|\text{det} \mathbf{T}'|(u,v,w) = abcw^2 \cos v.$]