Question 1 Solution

**Question 1** Find the domain and range of \( h(x) = \frac{2}{x^2-4} \).

**[Solution]**

For domain, the only undefined points for \( h(x) \) are those points, \( x \), with \( x^2 - 4 = 0 \). So, those points are \( \pm 2 \). Thus, the domain is

\[
x \in \mathbb{R} \text{ and } x \neq \pm 2
\]

or

\[
(-\infty, -2) \cup (-2, 2) \cup (2, \infty)
\]

or

\[
-\infty < x < -2 \text{ and } -2 < x < 2 \text{ and } 2 < x < \infty.
\]

For range, since \( h(x) \) is an even function, we have \( f(-x) = f(x) \) for all \( x \). Hence, to get range, we only need to consider \( x \geq 0 \). That is,

\[
0 \leq x < 2 \text{ and } 2 < x < \infty.
\]

Let \( y = \frac{2}{x^2-4} \). Then, since \( x \geq 0 \), we have

\[
x = \sqrt{\frac{2}{y} + 4}.
\]

Case I, when \( 2 < x < \infty \), we have

\[
2 < \sqrt{\frac{2}{y} + 4} < \infty.
\]

Thus,

\[
4 < \frac{2}{y} + 4 < \infty.
\]

So,

\[
0 < \frac{2}{y} < \infty.
\]

We have,

\[
0 < y < \infty.
\]
Case II, when $0 < x < 2$, we have

$$0 < \sqrt{\frac{2}{y} + 4} < 2.$$ 

Thus,

$$0 < \frac{2}{y} + 4 < 4.$$ 

So,

$$-4 < \frac{2}{y} < 0.$$ 

We have,

$$-2 < \frac{1}{y} < 0.$$ 

That is,

$$-\infty < y < -\frac{1}{2}.$$ 

Therefore, the range is

$$(-\infty, -\frac{1}{2}) \cup (0, \infty)$$

or

$$-\infty < y < -\frac{1}{2} \text{ and } 0 < y < \infty.$$