1. Compute the following limits if they exist:
   (a) \( \lim_{(x,y,z)\to(0,0,0)} \frac{x^2+3y^2}{x+1} \).
   (b) \( \lim_{(x,y)\to(0,0)} \frac{\sin 2x-2x+y}{x^3+y} \).

[Solution]
(a) Since \( \lim_{(x,y,z)\to(0,0,0)} x^2+3y^2 = 0 \) and \( \lim_{(x,y,z)\to(0,0,0)} x+1 = 1 \neq 0 \) in a neighbourhood of \((0,0,0)\), we use quotient property of limits to get
   \[ \lim_{(x,y,z)\to(0,0,0)} \frac{x^2+3y^2}{x+1} = \frac{0}{1} = 0. \]

(b) If we approaches \((0,0)\) along \(x = 0\), we have
   \[ \lim_{(0,y)\to(0,0)} \frac{\sin (2 \cdot 0) - 2 \cdot 0 + y}{0^3 + y} = \lim_{(0,y)\to(0,0)} \frac{y}{y} = 1. \]
   If we approaches \((0,0)\) along \(y = 0\), we have, by l’Hospital rule,
   \[ \lim_{(x,0)\to(0,0)} \frac{\sin 2x-2x}{x^3} = \lim_{x\to0} \frac{2 \cos 2x - 2}{3x^2} = \lim_{x\to0} \frac{-4 \sin 2x}{3 \cdot 2x} = -\frac{4}{3}. \]
   Therefore, the \( \lim_{(x,y)\to(0,0)} \frac{\sin 2x-2x+y}{x^3+y} \) does not exist.

2. Find an equation for the tangent plane of the graph of
   \[ f(x,y) = \frac{e^x}{x^2 + y^2} \]
   at the point \((1,2)\).

[Solution]
The equation of the tangent of the graph of \( f(x,y) \) is
   \[ z = f(x_0, y_0) + \left[ \frac{\partial f}{\partial x}(x_0, y_0) \right] (x-x_0) + \left[ \frac{\partial f}{\partial y}(x_0, y_0) \right] (y-y_0). \]
We have
\[ f(1, 2) = \frac{e^1}{1^2 + 2^2} = \frac{e}{5}; \]
\[ \frac{\partial f}{\partial x}(1, 2) = \frac{e^x (x^2 + y^2) - e^x (2x)}{(x^2 + y^2)^2} \bigg|_{(1, 2)} = \frac{e^1 (5) - e^1 (2 \cdot 1)}{(5)^2} = \frac{3e}{25}; \]
\[ \frac{\partial f}{\partial y}(1, 2) = \frac{0 (x^2 + y^2) - e^x (2y)}{(x^2 + y^2)^2} \bigg|_{(1, 2)} = \frac{-e^1 (2 \cdot 2)}{(5)^2} = -\frac{4e}{25}. \]

Therefore, the equation is
\[ z = \frac{e}{5} + \frac{3e}{25} (x - 1) - \frac{4e}{25} (y - 2) \]
\[ = \frac{e}{25} (3x - 4y + 10). \]

3. Compute the gradient of the function
\[ f(x, y, z) = \frac{x^2 + y}{z}. \]

[Solution]

The gradient of \( f \) is
\[ \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 2x & 1 & -\frac{x^2 + y}{z^2} \end{bmatrix}. \]

4. Sketch and describe the surface in \( \mathbb{R}^3 \) defined by the equation
\[ 4x^2 - 3y^2 + 2z^2 = 0. \]

[Solution]

Consider an equivalent equation \( 4x^2 + 2z^2 = 3y^2 \). When \( y = 0 \), the level curve is the origin. When \( y \neq 0 \), the section of the surface and the plane \( \{y = k\} \) is an ellipse centered the origin around the \( y \)-axis. The major axes are parallel to the \( z \)-axis and the minor axes are parallel to the \( x \)-axis. The ellipses get larger as \( |y| \) increases. When \( x = 0 \), the section is the straight line \( z = \pm \sqrt{\frac{3}{2}y} \). When \( z = 0 \), the section is the straight line \( x = \pm \sqrt{\frac{3}{4}y} \). Thus, the graph is two cones.
5. Suppose that a particle following the given path
\[ c(t) = (4e^t, 6t^4, \cos t) \]
flies off on a tangent at \( t = 0 \). Compute the position of the particle at time \( t = 1 \).

\[ \textbf{Solution} \]

The tangent line of \( c(t) \) at \( t = 0 \) is
\[
\begin{align*}
    l(t) &= c(0) + c'(0)(t - 0) \\
    &= (4, 0, 1) + \left(4e^t, 24t^3, -\sin t\right)_{t=0} \cdot t \\
    &= (4, 0, 1) + (4, 0, 0) \cdot t \\
    &= (4 + 4t, 0, 1).
\end{align*}
\]

When \( t = 1 \), the position of the particle is \( l(1) = (8, 0, 1) \).