Sample Midterm 2 100pts.

- There are 6 pages in the exam including this page.
- Write all your answers clearly. You have to show work to get points for your answers.
- You can write on both sides of the paper. Indicate that the answer follows on the back of the page.
- Use of Calculators is not allowed during the exam.

(1) ....../22
(2) ....../22
(3) ....../20
(4) ....../36
Total ....../100
(1) 22pts. Let $\mathcal{P}_2$ denote the set of all polynomials of degree less than or equal to 2. Let $T: \mathcal{P}_2 \to \mathcal{P}_2$ be a linear transformation defined as $T(f(t)) = f(t - 1)$, where $f(t)$ is a polynomial of degree less than or equal to 2.

(a) Find the matrix of this transformation with respect to the basis $\{1, t, t^2\}$ of $\mathcal{P}_2$. Show work.

(b) Evaluate the determinant of the matrix you found in part (a). Show work.
(2) 22 pts. Let \{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \} \) be a basis of the subspace \( W \) of \( \mathbb{R}^3 \).

(a) Find a orthonormal basis of \( W \). Show work.

(b) Find the orthogonal projection of the vector \( \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \) onto \( W \). Show work.
(3) 20pts. Let $\mathbb{C}$ denote the set of complex numbers $\{a + bi : a, b \in \mathbb{R}\}$. Then both $\mathcal{B}_1 = \{1, i\}$ and $\mathcal{B}_2 = \{1 + i, 1 - i\}$ are bases for $\mathbb{C}$.

(a) What is the matrix that transforms a vector in $\mathcal{B}_1$ coordinates into a matrix in $\mathcal{B}_2$-coordinates? Show work.

(b) Write down the element $4 + 2i$ in $\mathcal{B}_2$-coordinates. Show work.
(4) 16pts. Give short answers to the following.

(a) Let \( \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3 \). Compute \( \begin{vmatrix} a + d & b + e & c + f \\ 2g & 2h & 2i \\ d & e & f \end{vmatrix} \).

Give reasons to support your answer.

(b) Let \( A \) be a \( 2 \times 2 \) matrix such that \( \det A = -1 \) then \( A \) is orthogonal.

State true or false with justification.
(e) Let $u, v, w$ be vectors in $\mathbb{R}^n$. Let $w$ be orthogonal to both $u$ and $v$. Then $u + v$ is orthogonal to $3w$. State true or false with justification.