Linear System Example: Repeated Real Root

We solve Example 2 of §56 in [Simmons, Second edition], on p. 431–2,
\[
\begin{align*}
\frac{dx}{dt} &= 3x - 4y \\
\frac{dy}{dt} &= x - y
\end{align*}
\]
by an alternate method. We apply the Laplace transform with the generic initial conditions \(x(0) = k_1\) and \(y(0) = k_2\) to get
\[
\begin{align*}
pX - k_1 &= 3X - 4Y \\
pY - k_2 &= X - Y
\end{align*}
\]
or
\[
\begin{align*}
(3 - p)X - 4Y &= -k_1 \\
X - (1 + p)Y &= -k_2.
\end{align*}
\]
We solve these simultaneous linear equations for \(X\) and \(Y\) by elimination,
\[
\begin{align*}
[4 - (1 + p)(3 - p)]X &= -4k_2 + (1 + p)k_1 = (p + 1)k_1 - 4k_2 \\
[-(3 - p)(1 + p) + 1 \cdot 4]Y &= -(3 - p)k_2 + k_1 = k_1 + (p - 3)k_2.
\end{align*}
\]
The expressions in \([\ ]\) are both
\[
4 - (1 + p)(3 - p) = 4 - (3 + 2p - p^2) = p^2 - 2p + 1 = (p - 1)^2.
\]
We therefore divide out by this and express everything in terms of \(p - 1\), with an eye towards using the shift formula for Laplace transforms,
\[
\begin{align*}
X &= \frac{(p + 1)k_1 - 4k_2}{(p - 1)^2} = \frac{(p - 1)k_1 + 2k_1 - 4k_2}{(p - 1)^2} = \frac{k_1}{p - 1} + \frac{2k_1 - 4k_2}{(p - 1)^2} \\
Y &= \frac{k_1 + (p - 3)k_2}{(p - 1)^2} = \frac{(p - 1)k_2 + k_1 - 2k_2}{(p - 1)^2} = \frac{k_2}{p - 1} + \frac{k_1 - 2k_2}{(p - 1)^2}
\end{align*}
\]
Finally, we apply the inverse Laplace transform and find
\[
\begin{align*}
x &= k_1 e^t + (2k_1 - 4k_2)te^t = [k_1 + (2k_1 - 4k_2)t]e^t \\
y &= k_2 e^t + (k_1 - 2k_2)te^t = [k_2 + (k_1 - 2k_2)t]e^t.
\end{align*}
\]
This yields the solution (23) in the book if we take \(k_1 = 2\) and \(k_2 = 1\), and the solution (25) if we take \(k_1 = 1\) and \(k_2 = 0\). The correct values of \(k_1\) and \(k_2\) to use are obvious from the initial conditions. Moreover, we recover (26) if we put \(c_1 = k_2\) and \(c_2 = k_1 - 2k_2\).

Compare this treatment with the traditional one in the book to decide which is shorter or easier or more efficient. Draw your own conclusions.