Second Examination
Alternate Edition

80 points, 20 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Make sure your T.A.'s name is on each book, as well as your name.
Do not evaluate square roots, trigonometric functions and such.
Use only the officially provided blue books.

1. (a) Find all second order partial derivatives of the function \( f(x, y) = y \sin xy \) (this means \( y \sin(xy) \)).
   
   (b) Find all first order derivatives of the function \( z = g(x, y) \) defined implicitly by the equation
   
   \[ 3x^2y + 3xz + z^3 = 0. \]

2. Evaluate the double integral

\[ \int \int_{\Omega} xy \, dx \, dy, \]

where \( \Omega \) denotes the triangle with vertices \((0,0), (0,1), \) and \((2,0)\).

3. Use differentials to estimate the value of \( \sqrt{80} \sqrt{10} \), given that \( \sqrt{81} = 3 \) and \( \sqrt{9} = 3 \).

4. A rectangular box with an open top is to be constructed out of sheet material. The material for the four sides costs $3 per square foot, while the material for the bottom costs $2 per square foot. Find the dimensions of the most economical box that holds 27 cubic feet.