Math 202 Exam 1, Friday March 4, 2011.
Show all work in a clear and concise manner to get maximum credit. Circle your answers.

1. (15 points) Find all points on the curve \( \vec{c}(t) = < t, t^2, \frac{1}{3}t^3 > \) where the tangent line to the curve is parallel to the plane \( x + y + z = 0 \).

2a. (10 points) Find the area of the triangle with vertices \((1,0,0), (0,0,-2), (-1,-1,0)\).

2b. (10 points). Use Lagrange multipliers to find the point on the plane \( x - 2y - 2z = 1 \) that is closest to the point \((0,1,0)\).

3. (15 points) Let \( \hat{v} = \hat{i}, \hat{w} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} \). Suppose that \( f(x, y) \) is a differentiable function at \((0,0)\) with directional derivatives
   \[ D_{\hat{v}}f(0,0) = 2, \quad D_{\hat{w}}f(0,0) = 3. \]

   Find the directional derivative of \( f \) at \((0,0)\) in the direction of \( 3\hat{i} + 4\hat{j} \).
   Hint: Note that \( \hat{v} \) and \( \hat{w} \) are not orthogonal.

4. (15 points) Let \( f(x, y, z) = xyz \). At the point \( P = (1,2,3) \), in which unit direction is \( f \) decreasing the fastest? How far would you have to move in that direction so that \( f \) is approximately (linear approximation) 5.8?

5. (15 points) Let \( F(x, y, z) = x^3 + y^3 + z^3 + 6xyz + 4 \) and suppose that in a small neighborhood of the point \( P = (1, -1, 2) \) on the level surface \( \{(x, y, z) : F(x, y, z) = 0\} \), \( z \) can be implicitly expressed as a function of \( x \) and \( y \), i.e \( z = f(x, y) \).
   a. (8 points) Calculate \( f_x, f_y \) at \( P \) using the Chain rule.
   b. (7 points) Now suppose \( x = 2s - t, y = s - 2t^2. \) Again using the Chain rule, calculate \( \frac{\partial z}{\partial t}(1,1) \) when \( z = 2 \).

6. (20 points) Find the critical points of \( f(x, y) = x^3 + x^2y - y^2 - 4y \) and use the second derivative test to classify them (relative max, relative min, saddle, no information).