Calculus III PILOT Problem set 1, week of February 5

1. The center of mass of triangle ABC is the point G defined by
\[ \vec{GA} + \vec{GB} + \vec{GC} = 0. \]
Show that for any choice of origin O,
\[ \vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}. \]

2. Find a unit vector orthogonal to the line \( \vec{X}(t) = (-1, -1, 2) + t(2, -1, 1) \) and the vector \( \hat{i} - \hat{j} \).

3. Find the equation of the plane that contains (3,-1,2) and the line \( \vec{X}(t) = (2, -1, 0) + t(2, 3, 0) \).

4. Find the line through (3,1,-2) that intersects and is perpendicular to the line
\[ x = -1 + t, \quad y = -2 + t, \quad z = -1 + t. \]

5a. Let \( \vec{a} = e_1 - e_3 \) and \( \vec{c} = e_1 + 3e_2 + 2e_3 \). Find a vector \( \vec{b} \) such that \( \vec{a} \times \vec{b} = \vec{c} \) if one exists.

5b. Same problem for \( \vec{a} = 2e_1 - e_3 \) and \( \vec{c} = e_1 + 3e_2 + 2e_3 \).

6. Simplify
\[ i. (\vec{a} + \vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c}) \]
\[ ii. (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) \]
\[ iii. (\vec{a} + \vec{b} - \vec{c}) \times (\vec{a} - \vec{b} + \vec{c}) \]
\[ iv. (\vec{a} + \vec{b} - \vec{c}) \cdot (\vec{a} - \vec{b} + \vec{c}) \]