Calculus III PILOT Problem set 12, week of April 25

1. Let $S$ be the part of the parabola $z = 4 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$. Calculate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = <xy, yz, zx>$ and $S$ is oriented by the upward normal.

2. Let $\vec{F} = <-y^2, x, z^2>$ and let $C$ be the curve of intersection of the plane $y+z=2$ with the right circular cylinder $x^2+y^2 = 1$. Orient $C$ counterclockwise when viewed from above. Use Stoke’s theorem to evaluate $\int_C \vec{F} \cdot d\vec{s}$.

3. Let $\vec{F} = <y, z, x>$ and let $S$ be the part of the unit sphere (centered at the origin) in the positive octant oriented outward. Calculate $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$.

4. Let $\vec{V} = <1, 1, 1>$ and let $R$ be the solid region in the positive octant bounded by the unit sphere (centered at the origin) and the three coordinate planes. Orient the boundary of $R$ by the outward unit normal. Use the divergence theorem (Gauss theorem) to compute $\iint_S (x+y+z) \, dS$ where $S$ is as in problem 3. Use this to double check your answer in problem 3.

5. Let $\vec{F}$ and $S$ be as in problem 3 and let $C = \partial S$ oriented positively with respect to the orientation of $S$. Compute $\int_C \vec{F} \cdot d\vec{s}$. You should have verified Stoke’s theorem.

6. Consider the tetrahedron bounded by the plane $3x + y + 3z = 6$ and the coordinate planes and let $S$ be the piecewise smooth surface consisting of the three faces not in the $xz$ plane. Let $S$ be oriented by the normal pointing out of the tetrahedron. Use Stokes theorem to compute $\int \int_S \nabla \times \vec{F} \cdot d\vec{S}$ where $\vec{F} = <xz, -y, x^2y>$. 

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