Calculus III PILOT Problem set 2

1. Find \( \lim_{(x,y) \to (0,0)} \frac{\cos xy - 1}{x} \) or explain why the limit does not exist.

2. Verify that \( f(x, y, z) = e^{3x+4y} \cos (5z) \) satisfies
   \[
   f_{xx} + f_{yy} + f_{zz} = 0.
   \]

3. Find the points on the surface \( z = x^2 + 2xy - y^2 + 3x - 2y - 4 \) where the tangent plane is parallel to the \( xy \) plane.

4. Find the equation of the tangent plane to the implicitly defined graph \( z = z(x,y) \) defined by the relation \( x^3 + y^3 + z^3 + 3xyz = 2 \) at the point \((-1,1,2)\).

5. Find the equation of the tangent plane to the graph of \( z = x^y \) at the point \((1,1,1)\).

6. Let \( f(x, y, z) = xe^{yz} + ye^{xz} + ze^{xy} \). Find the directional derivative of \( f \) at the point \( P=(1,0,2) \) in the direction \( \vec{PQ} \), where \( Q=(5,3,3) \).

Remarks: A directional derivative of \( f(x, y, z) = f(\vec{X}) \) in the direction \( \vec{v} \) (always a unit vector) at a point \( Q \) is by definition

\[
D_{\vec{v}} f(Q) := \lim_{h \to 0} \frac{f(Q + h\vec{v}) - f(Q)}{h}.
\]

This is completely analogous to the definition of the partial derivative \( \frac{\partial f}{\partial x_i} \) where we choose \( \vec{v} = \vec{e}_i \) (where \( \vec{e}_i = (0, \ldots, 0, 1 \ (\text{the ith place}), 0, \ldots, 0) \)).