Calculus III PILOT Problem set 4

1. Consider the curve $\mathbf{c}(t) = (t^3 + 1, t^2 - 1, \frac{\sqrt{3}}{2} t^2)$, $0 \leq t \leq 1$. Reparametrize the curve in terms of arc length, i.e. find $\tilde{\mathbf{c}}(s) = c(t(s))$ and find the range of $s$, i.e. $0 \leq s \leq L$.

2. Find the terms up to second order of the Taylor expansion of $x^4 + x^2y^2 - y^4$ about the point $(1, 1)$. Verify your answer by finding (by direct computation) the exact Taylor expansion.

3. Find all the critical points of $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ and test for relative max, min and saddle points.

4. In $\mathbb{R}^3$, find the minimum distance from the origin to the surface of the cone $C = \{(x, y, z) : z^2 = (x - 1)^2 + (y - 2)^2\}$. Do not use Lagrange multipliers. Hint: minimize the distance squared $x^2 + y^2 + z^2$.

5. Redo problem 4 by the method of Lagrange multipliers.

6. Let $\mathbf{f}(t) : \mathbb{R} \rightarrow \mathbb{R}^3$ be a vector function satisfying $\mathbf{f}'(t) = \mathbf{f}(t)$, $\mathbf{f}(0) = 2(1, 1, 1)$. Find $\mathbf{f}(t)$.