Math 439 Exam 1, October 16, 2014
You may consult your textbook for this exam but not the internet. The exam is due in class Tuesday October 21.

1. (20 pts) Let $\gamma(s)$ be a unit speed curve in the plane $\mathbb{R}^2$. The curve $\gamma^\epsilon(s) = \gamma(s) + \epsilon n(s)$ ($n(s)$ is the unit normal vector of $\gamma$) is the parallel curve to $\gamma$ at distance $\epsilon$. Assume $|\epsilon \kappa(s)| < 1$ where $\kappa(s)$ is the signed curvature of $\gamma$. Show that $\gamma^\epsilon(s)$ is a regular curve and its signed curvature is

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\kappa^\epsilon(s) = \frac{\kappa(s)}{1 - \epsilon \kappa(s)}.
$$

2. Let $f : \mathbb{R}^2 \setminus \{0\} \to \mathbb{R}^2 \setminus \{0\}$ be defined by $f(p) = \frac{p}{|p|^2}$ where $p = (p_1, p_2)$, $|p| \neq 0$.
   a. (10pts) Compute $df_p(e_1)$ and $df_p(e_2)$ where $e_1, e_2$ is the standard basis of $\mathbb{R}^2$.
   b. (10pts) Show that the Jacobian determinant $J$ of the map $f$ is $J(p) = -\frac{1}{|p|^4}$.

3. (20 pts) Show that $S^2 \setminus (0, 0, 1)$ is diffeomorphic to the $x,y$ plane using stereographic projection. See Do Carmo p.67 problem 16.
