Math 421: Dynamical Systems & Chaos – Professor Haskins

Homework 3: 14 Feb 02
Due: 21 Feb 02 at start of class

Remember to show all your working and to write clearly, neatly.

Part I

From Devaney’s book “A First Course in Chaotic Dynamical Systems”:
Chapter 5: 1 – 9 (for questions 1 and 4 do every second part i.e. a, c, e, ..)

Part II: Comparing continuous and discrete time dynamical systems

1. For 1d discrete time dynamical systems we came up with a definition of the stability of a fixed point by looking at a linear approximation to our function near a fixed point. We looked at the behaviour of this linear dynamical system near its fixed point and came up with a criterion for a stable fixed point of our nonlinear dynamical systems in terms of the stability of the linearized system. We called the fixed point \( x_0 \) stable if \( |f'(x_0)| < 1 \), unstable if \( |f'(x_0)| > 1 \) and neutral if \( |f'(x_0)| = 1 \).

   In HW2 we saw how to define fixed points for a 1d continuous dynamical system \( \dot{x} = f(x) \) – they are the points for which \( f(x) = 0 \).
   (i) For the linear ODE \( \dot{x} = ax \) by using the explicit solution or otherwise determine for what values of \( a \) the origin is a stable, unstable or neutral fixed point.
   (ii) Using the same idea as in the discrete case (i.e., comparison with the linear approximation) come up with definitions of stable, unstable and neutral fixed points of a nonlinear ODE \( \dot{x} = f(x) \) in terms of the value of the derivative of \( f \) at the fixed point.

2. Let \( F_\lambda = \lambda x(1 - x) \).
   (i) Give a detailed description of the fixed points including their stability in the discrete dynamical associated with function \( F_\lambda \) (the logistic map).
   (ii) Using the criterion you developed in question 1 perform the same kind of fixed point analysis for the continuous dynamical system defined by \( F_\lambda \) (the logistic equation).
   (iii) Compare your answers to parts (i) and (ii) and draw some conclusions.