Quiz # 1

Exercise 1:
Let $G$ be the group of isometries preserving an equilateral triangle in the Euclidean plane.

1. List the elements of $G$; prove that your list is complete. What is the order of $G$?
2. Is $G$ commutative?
3. Is $G$ isomorphic to one of the groups $\mathbb{Z}/6\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$, or $S_3$? Prove your claims.

Exercise 2
Consider the upper half-plane model $H^2 = \{ z \in \mathbb{C} | \text{Im}(z) > 0 \}$ of the hyperbolic plane, with arclength element $ds = \frac{\sqrt{dx^2 + dy^2}}{y}$. Let $g$ be the isometry of $H^2$ given by the matrix

$$\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \in SL(2, \mathbb{R})$$

1. What is the distance $d$ between the points $p_1 = i$ and $p_2 = 2i$?
2. Find the images $g(p_1)$ and $g(p_2)$. What is the distance between these points?
3. Is $g$ an elliptic, parabolic, or hyperbolic isometry?
4. Find all fixed points of $g$ in $H^2$.
5. What is $g^2$? What does that tell us about $g$?