Exam 1: October 15, 2002    Time allowed: 85 minutes.

Maximum score 110 pts., but the exam is treated as a 100 pt. exam.

[10] 1. Let \( f(x, y) = \sin(2x^2 - xy^2) \). Determine \( f_x(0, 3) \) and \( f_y(x, y) \).

[5] 2. Determine the angle between the vectors \( 3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \) and \( 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} \).

[5] 3. Express in simplest form: \( [(\sin t)(\mathbf{i} + t^2\mathbf{j})] \times e^{3t}\mathbf{i} \).

[3,12] 4. Let \( \ell \) be the line with equations

\[
\frac{x}{2} = \frac{y - 1}{3} = \frac{z + 1}{4}.
\]

a) Determine that \( \ell \) does not pass through the origin.

b) Determine the equation of the plane that passes through the origin and contains the line \( \ell \).

[10] 5. Sketch the \( c \)-level curves of the surface \( x^2z^3 - yz^3 = 1 \) for \( c = 0, 1, \frac{1}{2}, 2 \). As customary, the \( z \)-coordinate gives the level.

[20] 6. A function of two variables is given by the formula:

\[
f(x, y) = \begin{cases} 
  y & \text{if } y > 0 \\
  x^2 & \text{if } y = 0 \\
  0 & \text{if } y < 0
\end{cases}
\]

a) Verify that \( f_x(x, y) \) exists for all \( (x, y) \).

b) Determine the domain of \( f_y \).

[5,10,5] 7. Let \( \ell_1 \) be the line with parametric equations

\[
x = t, \quad y = 2t, \quad z = 5 - t.
\]

and \( \ell_2 \) be the line with parametric equations

\[
x = t + 1, \quad y = t + 2, \quad z = t + 3.
\]

a) Verify that \( \ell_1 \) and \( \ell_2 \) are not parallel.

b) Verify that \( \ell_1 \) and \( \ell_2 \) do not intersect.

c) We project \( \ell_1 \) and \( \ell_2 \) onto the \( (x, y) \)-plane. Determine the point of intersection of their projections.

[10,5] 8. A particle moves in the plane according to parametric equations:

\[
\mathbf{r}(t) = t^3\mathbf{i} + t^5\mathbf{j} + t^7\mathbf{k}.
\]

a) Determine equations (any kind) for the line tangent to the curve at the point \((1,1,1)\).

b) Express the length of the segment of the curve from the origin to \((1,1,1)\) as a definite integral with explicit integrand and limits of integration.

[10] 9. Let \( g(x, y) = xy(x^2 + y^4)^{-1} \). Compute the limit of \( g(x, y) \) as \( (x, y) \to (0, 0) \) along the line \( x = 2y \).