Why use radian measure for angles?

You may be wondering why one wants to discard the degree as the unit of angle measurement. Here’s the best reason, though it may take time for you to grasp:

First, draw for yourself a large picture of the unit circle in the \((x, y)\)-plane.

Bigger!

On that picture, mark a small angle of \(\theta\) radians in standard position, so that the sides of the angle intersect the unit circle. The whole point of radian measure is that the size of your angle in radians is the length of the arc on the unit circle that the angle cuts out. (Thus, a right angle is a quarter of \(2\pi\), i.e., \(\frac{\pi}{2}\).) Label that arc as \(\theta\).

Drop the perpendicular from the point of the unit circle on the terminal side of our angle to the \(x\)-axis. By our definitions, the length of that line segment is \(\sin \theta\).

You should now have two lengths marked: the arc cut out by the angle and the straight line segment dropped from the terminal point. How do the two lengths compare as \(\theta\) gets smaller and smaller? This is the very question that underlies the calculus of the trig functions (in Calculus I).

Please try to give the answer to this question, on an intuitive basis.

The answer: while it’s always the case (for acute angles) that \(\sin \theta < \theta\), their ratio, i.e., \(\sin \theta / \theta\) goes to 1 as \(\theta\) goes to 0. This is the reason we prefer radians; otherwise, there would be an annoying factor of \(\pi/180\) every time we had to use calculus on the trig functions. For those who are in the know (and those who wish to be in the know), the fact that the ratio goes to 1, plus the fact that \(\sin 0 = 0\), is why the Taylor Polynomials for \(\sin x\) begin with \(x\).