4. The speed $v$ of blood flowing along the central axis of an artery of radius $R$ is given by Poiseuille’s law

$$v(R) = cR^2$$

where $c$ is a constant. You want to determine the speed to within an accuracy of 12%. Use the tangent line to the graph of $v$ at $R_0$ to estimate to within what accuracy you should measure the radius.

**Solution:** Let $R$ be our measurement of the radius, and let $R_0$ be the actual radius. Then the percent accuracy to which we have measured the radius is

$$\frac{|R - R_0|}{|R_0|}$$

and the percent accuracy to which we have measured the velocity is

$$\frac{|v(R) - v(R_0)|}{|R_0|}$$

We want this second quantity to be 0.12. Let’s begin by simplifying it. By using the tangent line to the graph of $v$ at $R_0$, we can approximate $v(R)$:

$$v(R) \approx v(R_0) + v'(R_0)(R - R_0)$$

so that

$$v(R) - v(R_0) \approx v'(R_0)(R - R_0)$$

Plugging this in above, we get

$$\approx \frac{|v'(R_0)||R - R_0|}{v(R_0)}$$

Now, we know that $v(R) = cR^2$ so that $v(R_0) = cR_0^2$ and $v'(R_0) = 2cR_0$. Plugging this in, we get

$$\frac{|2cR_0||R - R_0|}{cR_0^2} = 2\frac{|R - R_0|}{|R_0|}$$

So, if we want to measure the velocity to 12% percent accuracy, we should measure the radius to $12%/2 = 6\%$ accuracy.

5. You run a very specialized business that only sells Robinson R22 helicopters. These helicopters cost 7 million dollars to build. If you sell a shipment of $x$ helicopters, you can sell each helicopter in that shipment for $f(x) = 10 - \sqrt{x}$ million dollars. How many helicopters should you sell to maximize profit?

**Solutions:** If we sell $x$ helicopters, we make $f(x)$ million dollars for each helicopter, so we will make $xf(x)$ dollars total from selling $x$ helicopters. However, we have also spent 7 million dollars on each helicopter, so building $x$ helicopters costs 7$x$ million dollars. So our total profit is the difference between these two:

$$P(x) = xf(x) - 7x = x(10 - \sqrt{x}) - 7x = 3x - x^{3/2}$$
This is the function that we want to maximize. So we solve the equation $P(x) = 0$ to find possible maxima:

$$P'(x) = 3 - \frac{3}{2}\sqrt{x} = 0$$

$$3 = \frac{3}{2}\sqrt{x}$$

$$2 = \sqrt{x}$$

$$x = 4$$

To check that this critical point is a local maximum, we compute the second derivative:

$$P''(x) = -\frac{3}{4\sqrt{x}}$$

$$P''(4) = -\frac{3}{4\sqrt{4}} = -\frac{3}{8} < 0$$

Since the second derivative is negative, there is in fact a maximum at $x = 4$ (the slope of the tangent line is decreasing-$P(x)$ is concave down at $x = 4$). Thus, to maximize profit, we should sell 4 helicopters.