Solutions for Diff.Eq. Homework 5

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1 Section 3.6

In each of problems find the general solution of the given differential equation.

Problem 1: $y'' - 2y' - 3y = 3e^{2t}$

Solution: First we find the general solution of the homogeneous equation, we have

$$y = c_1 e^{3t} + c_2 e^{-t}$$

Next we find a special solution, let $Y = Ae^{2t}$, plug $Y$ in the equation, we have

$$Y = -e^{2t}$$

Hence the general solution is

$$y = c_1 e^{3t} + c_2 e^{-t} - e^{2t}$$

Q.E.D.

Problem 2: $y'' + 2y' + 5y = 3\sin 2t$

Solution: First we find the general solution of the homogeneous equation, we have

$$y = c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t$$

Next we find a special solution, let $Y = A \cos 2t + B \sin 2t$, plug $Y$ in the equation, we have

$$Y = \frac{3}{17} \sin 2t - \frac{12}{17} \cos 2t$$

Hence the general solution is

$$y = c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t + \frac{3}{17} \sin 2t - \frac{12}{17} \cos 2t$$

Q.E.D.
Problem 3: \( y'' - 2y' - 3y = -3te^{-t} \)

Solution: First we find the general solution of the homogeneous equation, we have

\[ y = c_1e^{3t} + c_2e^{-t} \]

Next we find a special solution, let \( Y = At e^{-t} + Bt^2 e^{-t} \), plug \( Y \) in the equation, we have

\[ Y = \frac{3}{8}t^2 e^{-t} + \frac{3}{16}te^{-t} \]

Hence the general solution is

\[ y = c_1e^{3t} + c_2e^{-t} + \frac{3}{8}t^2 e^{-t} + \frac{3}{16}te^{-t} \]

Q.E.D.

Problem 6: \( y'' + 2y' + y = 2e^{-t} \)

Solution: First we find the general solution of the homogeneous equation, we have

\[ y = c_1e^{-t} + c_2te^{-t} \]

Next we find a special solution, let \( Y = At^2 e^{-t} \), plug \( Y \) in the equation, we have

\[ Y = t^2 e^{-t} \]

Hence the general solution is

\[ y = c_1e^{-t} + c_2te^{-t} + t^2 e^{-t} \]

Q.E.D.

Problem 9: \( y'' + \omega^2_0 y = \cos \omega t, \quad \omega^2 \neq \omega^2_0 \)

Solution: First we find the general solution of the homogeneous equation, we have

\[ y = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t \]

Next we find a special solution, let \( Y = A \cos \omega t + B \sin \omega t \), plug \( Y \) in the equation, we have

\[ Y = \frac{1}{\omega_0^2 - \omega^2} \cos \omega t \]

Hence the general solution is

\[ y = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t + \frac{1}{\omega_0^2 - \omega^2} \cos \omega t \]

Q.E.D.
Problem 10: $y'' + \omega_0^2 y = \cos \omega_0 t$

Solution: First we find the general solution of the homogeneous equation, we have

$$y = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

Next we find a special solution, let $Y = At \cos \omega_0 t + Bt \sin \omega_0 t$, plug $Y$ in the equation, we have

$$Y = \frac{t}{2\omega_0} \sin \omega_0 t$$

Hence the general solution is

$$y = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t + \frac{t}{2\omega_0} \sin \omega_0 t$$

Q.E.D.